Practice Problems for Midterm 2, Math 23B, Dr. Frank B"auerle

Note: These practice problems provide you with examples of the types of problems you might encounter on your midterm. This list is longer than your actual midterm, and NOT exhaustive of the type of problem you might encounter. Solutions will be posted later on the class web site. There will be review on Wednesday in class with Frank and in your sections.

1. Let \( \vec{c}(t) = (\cos^3 t, \sin^3 t) \) where \( 0 \leq t \leq 2\pi \). Evaluate \( \int_{c(t)} y \, dx + x \, dy \)

2. Identify the following integrals as either a path, line, scalar surface or vector surface integral. Give a brief description what these integrals might represent. Assume that all integrals are well defined, so for instance \( \vec{F} = (F_1, F_2, F_3) \) is a continuous vector field, \( S \) represents a regular surface, etc. etc.

(a) \( \int_{\Omega(t)} F_1 \, dx + F_2 \, dy + F_3 \, dz \),
(b) \( \int_{\Omega(t)} f(x, y, z) \, ds \),
(c) \( \int_D (F_1 \cdot \frac{\partial g}{\partial y} + F_2 \cdot \frac{\partial g}{\partial z} + F_3) \, dA \),
(d) \( \int_{\Omega(t)} \vec{F} \cdot d\vec{s} \),
(e) \( \int_S \vec{F} \cdot d\vec{S} \),
(f) \( \int_D f(x,y,g(x,y)) \, dA \),
(g) \( \int_D ||\vec{T}_u \times \vec{T}_v|| \, dA \)

3. Evaluate \( \int_{c(t)} \vec{F} \cdot d\vec{s} \) where \( \vec{F}(x, y) = (-2y - 2x, 3x + 2y) \) and \( C \) is the curve (ellipse) given by \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \) oriented counter-clockwise.

4. Evaluate \( \int_{c(t)} x \, dx + y \, dy + z^2 \, dz \) where \( \vec{c}(t) = (\cos t, \sin t, t) \) and \( 0 \leq t \leq 2\pi \).

5. Evaluate \( \int_S \vec{F} \cdot d\vec{S} \) where \( \vec{F}(x, y, z) = (x, y, 1) \) and \( S \) is the upper hemisphere \( x^2 + y^2 + z^2 = 1 \), \( z \geq 0 \) oriented with the outer normal.

6. Evaluate \( \int_C f \, ds \) where \( f(x, y) = y \cos(2\pi x) \) and \( C \) is the boundary of the triangle with sides \( x = 0, y = 0, x + y = 1 \).

7. Evaluate \( \int_S 6yz \, dS \) where \( S \) is the portion of the plane \( x + y + z = 1 \) that is in the first octant. Does the orientation of \( S \) matter?

8. Let \( \Phi(u, v) = (u - v, u + v, uv) \) be a mapping from the unit disk in the \( u, v \)-plane onto a surface \( S = \Phi(D) \) in \( x, y, z \)-space. Find the area of \( S \).

9. Express the length of the curve \( y = e^{-x} \sin x \) for \( 0 \leq x \leq \pi \) as an integral. Don’t evaluate.

10. Integrate \( f(x, y, z) = y^2 \sin(x^3) + e^{-x^2}y^2z \) over the quarter-circle \( z = \sqrt{1-y^2} \) from \( (0,1,0) \) to \( (0,0,1) \).

11. Let \( \Phi(u, v) = (e^u \cos v, e^u \sin v, v) \) be a mapping from \( D = [0, 1] \times [0, \pi] \) in the \( u, v \)-plane onto a surface \( S = \Phi(D) \) in \( x, y, z \)-space.

   (a) Find \( \vec{T}_u \times \vec{T}_v \)
   (b) Find an equation for the tangent plane to \( S \) when \( (u, v) = (0, \frac{\pi}{2}) \).
   (c) Find the area of \( S \).

12. Let \( \vec{F} = (z \cos x \cos y)\vec{i} + (2y^3 - z \sin x \sin y)\vec{j} + (3y^2z^2 + \sin x \cos y)\vec{k} \).

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(a) Let \( C \) be the curve of intersection of the ellipsoid \( \frac{x^2}{4} + \frac{y^2}{3} + \frac{z^2}{2} = 1 \) and the plane \( x + y + z = 1 \). Compute \( \int_C \vec{F} \cdot d\vec{s} \).

(b) Thinking of \( \vec{F} \) as a force field, compute the work done by \( \vec{F} \) on a particle that is moving from the lowest point to the highest point on the ellipsoid given in (a). (Positive z-direction is up)

(c) Let \( \vec{c}(t) = (t \cos t, t \sin t, te^{t^2}) \) where \( 0 \leq t \leq 4\pi \). Compute \( \int_C \vec{F} \cdot d\vec{s} \).

13. Let \( S \) be the surface \( x^2 + 2y^2 + z^2 = 1 \). Find a parametrization of \( S \) and use it to find the tangent plane to \( S \) at the point \( (-\frac{1}{\sqrt{2}}, \frac{1}{2}, 0) \).

14. Evaluate \( \int_C \sin z \, dx + \cos(\sqrt{y}) \, dy + x^3 \, dz \) where \( C \) is the line segment joining \((1, 0, 0)\) to \((0, 0, 3)\).

15. Let \( S \) be the surface defined to be the portion of the plane \( z + x + y = -1 \) where \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \) with the normal pointing upward. Find the flux of the vector field \( \vec{F} = x\vec{i} - 2y\vec{j} + xz\vec{k} \) across the surface \( S \).

16. Evaluate \( \int_S \vec{F} \cdot d\vec{S} \) where \( \vec{F} = (x, y, -y) \) and \( S \) is the cylindrical surface defined by \( x^2 + y^2 = 1, 0 \leq z \leq 1 \) with the normal pointing out of the cylinder.