Notes: Show your work. In other words, just writing the answer, even if correct, may not be sufficient for full credit. Scientific calculators are allowed, but no programmable and/or graphing calculators. And please put away your cell phones and other electronic devices, turned off or in airplane mode.

Your Name: ____________________________

Problem 1: out of 20
Problem 2: out of 20
Problem 3: out of 20
Problem 4: out of 20
Problem 5: out of 20

Total: out of 100

Good luck and have a relaxed weekend!
1. (a) (20 points) Assume $D$ is the triangular region in the $(x, y)$-plane with vertices $(0, 0), (2, 0)$ and $(0, 2)$. Compute
\[ \iint_D x - y^2 \, dA \]
2. (20 points) Compute the following integral. Hint: Draw the region of integration and change the order of integration.

\[
\int_{y=0}^{y=4} \int_{x=\sqrt{y}}^{x=2} \sqrt{x^3 + 1} \, dx \, dy
\]
3. (20 points) Let $W$ be the sphere of radius $R > 0$ centered at the origin.

(a) Use a triple integral to verify the formula for the volume of $W$.

Hint: what should the integrand be so that the triple integral computes the volume of $W$?

(b) Let $a, b, c$ be positive real numbers. Compute the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$

Hint: use a suitable change of variables to change the region of integration to the unit sphere and use your result from (a).
4. (20 points) Let the solid region \( W \subseteq \mathbb{R}^3 \) lie between the cylinder \( x^2 + y^2 = 4 \) and the planes \( z = 1 \) and \( z = 4 \). Assume that the mass density at a point \( P(x, y, z) \) in \( W \) is given by
\[
\delta(x, y, z) = z e^{x^2 + y^2}
\]

(a) Find the mass of the solid \( W \).

(b) Compute the coordinates of the center of mass of \( W \).
   
   Hint: two of the three coordinates of the center of mass are easy to find (but justify your answer).
5. (20 points) Assume $D$ is the parallelogram in the $(x, y)$-plane with vertices $(0, 0), (1, 1), (2, 0)$ and $(3, 1)$ and $f(x, y)$ is integrable over $D$.

(a) Let $D^* = [0, 1] \times [0, 1]$. Find a linear transformation $T : D^* \rightarrow D$ that maps $D^*$ onto $D$. Draw pictures!

(b) Compute the average value of the function $f(x, y) = \frac{x - y}{1 + y^2}$ on $D$. 