Notes: Show your work. In other words, just writing the answer, even if correct, may not be sufficient for full credit. Scientific calculators are allowed, but no programmable and/or graphing calculators. And please put away your cell phones and other electronic devices, turned off or in airplane mode.

Your Name: ____________________________

Problem 1: out of 20
Problem 2: out of 20
Problem 3: out of 20
Problem 4: out of 20
Problem 5: out of 20

Total: out of 100

Good luck and have a relaxed weekend!
1. (20 points) Let \( \vec{r}(t) = (\sin 2t, \cos 2t, t^2) \) represent the position of a particle as a function of time.

(a) Find the velocity vector \( \vec{v}(t) = \vec{r}'(t) \) of the particle.

(b) Find the speed of the particle when \( t = \pi \).

(c) Find the equation of the tangent line to the particle’s path at \( t = \pi \).

(d) Is there a time \( t \) when the particle is at rest? Explain your answer, i.e. correct answer with no explanation is worth one point.
2. (20 points) Consider the ellipsoid given by

\[ \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1 \]

(a) Find a unit normal vector to the ellipsoid at the point \((1, \frac{3}{\sqrt{2}}, 2)\).

(b) Find an equation of the tangent plane to the ellipsoid at the point \((1, \frac{3}{\sqrt{2}}, 2)\).
3. (20 points) Let
\[ f(x, y) = (x^2y, x^4 + 2y, 3e^{x-y}) \]
and
\[ g(r, \theta) = (r^3 \cos 2\theta, r^3 \sin 2\theta) \]
Find \( D(f \circ g)(1, \pi) \).
4. (20 points) Assume the temperature at a point \((x, y, z)\) is given by the function

\[ T(x, y, z) = x y^2 + y^2 z^3 + z^3 x \]

(a) From the point \(P(1, -2, 0)\), in what direction will temperature increase the most?

(b) Compute the directional derivative of \(T(x, y, z)\) at the point \(P(1, -2, 0)\) in the direction of the vector \((-1, 2, 3)\).

(c) From the point \(P(1, -2, 0)\), find a direction where temperature will not change at all.
5. (20 points) Find and classify all critical points for

\[ f(x, y) = 1 + xy^2 + 2x^2 + y^2 \]