1. (70 points) Evaluate the following integrals:

(a) (10 points) $\int_0^\pi \sin x \, dx$

(b) (10 points) $\int_0^e \frac{\ln x}{x} \, dx$

(c) (10 points) $\int \frac{x}{(x - 1)(x + 2)} \, dx$
(d) (10 points) \( \int_0^1 x \sin(x) \, dx \)

(e) (10 points) \( \int_0^{\frac{\pi}{4}} \frac{x^2 + 2x + 1}{1 + x^2} \, dx \)
(f) (10 points) \( \int \tan^5 \theta \sec^4 \theta \, d\theta \)

(g) (10 points) \( \int_{-\pi}^{\pi} \sin^3 x \, dx \)
(h) (Extra Credit, 10 points) \( \int \frac{1}{1 - \cos t} \, dt \)

2. (10 points) A 200 pound cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?

3. (10 points) Using integration, verify the formula \( V = \frac{1}{3} \pi r^2 h \) for the volume of a right circular cone with radius \( r \) and height \( h \).
4. (20 points) Three math majors have ordered a circular pizza with a 14 inch diameter. Instead of slicing it in the traditional way, they decide to slice it by two straight, parallel cuts. Where do the cuts have to be that all students get the same amount? Set up but do not evaluate an integral to compute the locations of the cuts.
5. (30 points) Let $\mathcal{R}$ be the region bounded by $y = 1 - \sin x$, $y = 0$, $x = 0$ and $x = \pi$. Set up, **BUT DO NOT EVALUATE**, integrals that compute the following. (Hint: It may be helpful to draw pictures.) You can use any method you like.

(a) The volume of the solid generated by rotating $\mathcal{R}$ about the $x$-axis.

(b) The volume of the solid generated by rotating $\mathcal{R}$ about the axis $y = 2$.

(c) The volume of the solid generated by rotating $\mathcal{R}$ about the $y$-axis.

(d) The length of the curve $y = 1 - \sin x$ between $x = 0$ and $x = \pi$.

(e) The surface area of the solid obtained in (a).
6. (20 points) Determine which of the following integrals converges/diverges. Give reasons for your answers. Compute the values of the converging integrals.

(a) \[ \int_{0}^{1} \frac{1}{\sqrt{x}} \, dx \]

(b) \[ \int_{-\infty}^{\infty} \frac{2x}{1 + x^2} \, dx \]
7. (20 points) Test for absolute convergence, conditional convergence and divergence for the following series. Indicate which test you are using and explain why it works.

(a) \( \sum_{n=1}^{\infty} \frac{n^2 + 1}{\sqrt{n^4 + 8n^2}} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \)

(c) \( \sum_{n=1}^{\infty} \frac{2^n n^4}{n!} \)
8. (20 points) Give the power series expansion and the radius of convergence for the following functions. Hint: you can (and should) use all knowledge about geometric series and how to differentiate, integrate and do arithmetic with power series rather than use the direct Taylor series approach.

(a) \( f(x) = \frac{x^2}{2+x} \)

(b) \( f(x) = \ln(1 + x) \)
9. (20 points) Find the radius and interval of convergence of the following series:

(a) \[ \sum_{n=1}^{\infty} \frac{(x - 3)^n}{n} \]

(b) \[ \sum_{n=1}^{\infty} \frac{x^{2n}}{n4^n} \]
10. (15 points) A rubber ball is dropped from $h$ ft. After each bounce it returns two thirds of the height of the previous bounce. What is the total distance the ball travels up and down?

11. (10 points) Use the Taylor polynomial of order three for $e^x$ centered at zero to estimate $\sqrt{e}$. What is the maximum error in this estimate? Compare your estimate to the value your calculator gives you.
12. (20 points)

(a) Find the first five terms of the McLaurin series expansion of \( f(x) = e^{x^2} \) centered at \( a = 0 \). Give the radius and interval of convergence.

(b) Find the first five terms of the Taylor series expansion of \( f(x) = x^2 \sin x \) centered at \( a = \frac{\pi}{4} \). Give the radius and interval of convergence.