Practice Problems for Midterm I, Math 19B, Bäuerle

1. Let $R$ be the triangle bounded by $y = 3x, y = 6, x = 0$. Compute the volume of the solid generated by rotating $R$ about the $y$-axis and the $x$-axis.

2. Find the area bounded by the curves $y = x^4$ and $y = 16x$.

3. Assume $f(x)$ and $g(x)$ are two continuous functions such that $\frac{df}{dx} = g(x), f(7) = 10$ and $f(3) = 1$. Find $\int_3^7 g(x)\,dx$.

4. Evaluate the following integrals:
   \[
   \begin{align*}
   &\int_1^e \frac{dx}{x(ln \ x + 1)^6} \quad \int_0^\frac{\pi}{4} \sec^2(3x)e^{\tan(3x)} \, dx \quad \int_0^e e^x \, dx \\
   &\int \frac{1}{4 + x^2} \, dx \quad \int e \frac{e}{e + 1} \, dx \quad \int_{-4}^4 t \cos t \, dt
   \end{align*}
   \]

5. Set up integrals (but do NOT evaluate) that give the volume of the solids generated by rotating the given region about the indicated axis.
   (a) Region in the first quadrant bounded by $y = 1 - x^2, y = 1$ and $y = \ln x$. Axis is $x = 0$
   (b) Region bounded by $y = x^2 + 2x, y = x, x = 1$ and $x = 2$. Axis is $y = -1$.
   (c) Region in the first quadrant bounded by $y = x^\frac{1}{3}$ and $y = x^2$. Axis is $x = -1$.

6. The velocity function of a particle moving along a line is given as $v(t) = t^2 - 2t - 8$ for $1 \leq t \leq 6$. Find the displacement of the particle and the total distance traveled by the particle.

7. A drill of radius one bores a hole through the center of a sphere of radius four. What is the remaining volume?

8. A raindrop is falling from the sky. After 20 seconds its velocity is 20 ft/sec. After how many seconds will its velocity be 100 ft/sec? Ignore air resistance.

9. Find $a$ such that the line $x = a$ cuts the area bounded by $y = \sin x, y = 0, x = \frac{\pi}{2}$ in half.

10. Let $f(x) = x^2 - 1$ in this problem.
   (a) Use a regular partition to set up (and do NOT evaluate unless you have nothing better to do) a Riemann sum for $\int_1^4 f(x) \, dx$ using the right endpoint as $x_i^*$ of each subinterval.
   (b) Compute this Riemann sum for $n = 4$.
   (c) Compute $\int_1^4 f(x) \, dx$ via antidifferentiation.
   (d) Sketch the graph of the function and interpret the value of the definite integral in terms of area(s).

11. (a) Compute the slope of the tangent line to $f(x) = \int_1^{x^2} \frac{1}{1 + t^3} \, dt$ at $x = 2$.
   (b) Find a function $f$ and a constant $a$ so that $\int_a^x f(t) \, dt = e^{x^2} - 1$.