Notes: Scientific calculators are allowed, but no programmable and/or graphing calculators. And please put away your cell phones and other electronic devices, turned off or in airplane mode. Put your answers in the provided boxes.

Your Name: ___________________________ Your Score: _____ Last Initial: _____

1. (30 points) Evaluate the following integrals.

(a) \[ \int_{0}^{\frac{\pi}{2}} (\cos x)e^{\sin x} \, dx \]

(b) \[ \int x \sin x \, dx \]

(c) \[ \int_{0}^{\frac{\pi}{4}} \tan^3(\theta) \sec^2(\theta) \, d\theta \]

2. (20 points) For this problem let \( R \) be the region between the \( x \)-axis and the graph of \( f(x) = \sin(x^2) \) where \( 0 \leq x \leq \sqrt{\pi} \).

(a) (5 points) Set up (but do NOT evaluate) an integral for the volume of the solid generated by rotating \( R \) about the \( x \)-axis.

(b) (5 points) Set up (but do NOT evaluate) an integral for the volume of the solid generated by rotating \( R \) about the \( y \)-axis.

(c) (10 points) Now evaluate your integral in (b) to find the volume of the solid generated by rotating \( R \) about the \( y \)-axis.

Test continues on the next page!
3. (10 points) The following improper integral converges. Find its (finite) value.

\[ \int_3^\infty \frac{1}{x^2} \, dx \]

4. (10 points) Use a suitable trig substitution to evaluate \( \int \frac{x^2}{\sqrt{4 - x^2}} \, dx \).

Substitution:

Evaluate:

Answer =

5. (10 points) Evaluate \( \int \frac{6}{x(x-1)(x-3)} \, dx \) after giving the completed partial fraction decomposition (PFD) of the integrand (i.e. include the values of the constants in the numerators).

PFD =

Answer =

6. (10 points) Test the following series for absolute convergence, conditional convergence and divergence and circle Y for Yes or N for No for each of the possibilities in the box below. Also indicate which test(s) you are using.

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + n} \)

Divergent: Y/N
Convergent: Y/N
Absolutely convergent: Y/N
Test(s) used: (Circle one or more)
Comparison/Integral/AlternatingSeries

(b) \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} \)

Divergent: Y/N
Convergent: Y/N
Absolutely convergent: Y/N
Test(s) used: (Circle one or more)
Comparison/Integral/AlternatingSeries

Test continues on the next page!
7. (10 points) Find the radius and the interval of convergence of \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x - 1)^n}{n^22^n} \)

Radius = \[ \underline{\text{ }} \]  
Interval = \[ \underline{\text{ }} \]

8. (20 points) (a) Find the third-degree Taylor polynomial centered at \( a = 0 \) for the function \( f(x) = e^x \).

\[ \underline{\text{ }} \]

(b) Find the third-degree Taylor polynomial centered at \( a = 1 \) for \( f(x) = e^x \).

\[ \underline{\text{ }} \]

(c) Find the Taylor Series centered at \( a = 0 \) for the function \( f(x) = e^x \). Give your answer in summation (\( \Sigma \)) notation.

\[ \text{Series (in } \Sigma \text{ notation)} = \underline{\text{ }} \]

(d) Find the Taylor Series centered at \( a = 0 \) for the function \( f(x) = xe^x \). Give your answer in summation (\( \Sigma \)) notation.

\[ \text{Series (in } \Sigma \text{ notation)} = \underline{\text{ }} \]
9. Answers

Question 1: (a) \( e - 1 \), (b) \(-x \cos x + \sin x + C\) (c) \( \frac{1}{4} \)

Question 2: (a) \( \pi \int_{0}^{\sqrt{\pi}} \sin(x^2) \, dx \), (b) \( 2\pi \int_{0}^{\sqrt{\pi}} x \sin(x^2) \, dx \) (c) Volume = \( 2\pi \)

Question 3: \( \frac{1}{3} \)

Question 4: Substitution : \( x = 2 \sin(\theta) \), Integral after substitution= \( \int 4 \sin^2 \theta \, d\theta \), Answer = \( 2\sin^{-1}(\frac{x}{2}) - \frac{1}{2}x\sqrt{4-x^2} + C \)

Question 5: PFD = \( \frac{2}{x} - \frac{3}{x-1} + \frac{1}{x-3} \) Integral = \( 2\ln|x| - 3\ln|x-1| + \ln|x-3| + C \)

Question 6: (a) N,Y,Y, Comparison Test (b) Y,N,N, Integral Test.

Question 7: \( R = 2 \), Interval = \([-1, 3]\)

Question 8: (a) \( 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \) (b) \( e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 \) (c) \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \) (d) \( \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} \)