Notes: Show your work. In other words, just writing the answer, even if correct, may not be sufficient for full credit. Scientific calculators are allowed, but no programmable and/or graphing calculators. And please put away your cell phones and other electronic devices, turned off or in airplane mode.

Your Name: ________________________________

Problem 1: out of 25
Problem 2: out of 25
Problem 3: out of 25
Problem 4: out of 25
Problem 5: out of 25
Problem 6: out of 25
Problem 7: out of 25
Problem 8: out of 25

Total: out of 200

Good luck and have a great Spring break!
1. (25 points) Find and classify all critical points for

\[ f(x, y) = xy - x^2y - xy^2 \]
2. (25 points) Let $T(x, y, z) = e^{-x^2-2y^2+3z^2}$ correspond to the temperature in a room, measured in degrees Celsius. Assume spatial coordinates are measured in meters.

(a) (10 points) Find the directional derivative of $T$ at the point $(1,1,1)$ in the direction of the vector $(2, -1, -2)$.

(b) What is the maximal directional derivative at the point $(1,1,1)$?

(c) (5 points) Give a direction you can move from $(1,1,1)$ so that temperature does not change.

(d) (5 points) Describe all directions you can move from $(1,1,1)$ so that temperature changes at a rate of two degrees Fahrenheit per meter.
3. (25 points) Find the first and second order Taylor approximations for the function \( f(x, y) = \sin(xy) \) at the point \((x_0, y_0) = (\frac{\pi}{6}, 1)\). You can give your answer either as \( T(x, y) \) or \( T(h_1, h_2) \).
4. (25 points) Assume you are given the surface $S$ with equation

$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

(a) (2 points) Classify the given surface. (Sketch it or give its name).

(b) (10 points) Find the equation of the tangent plane to the surface $S$ at the point $(\sqrt{3}, \sqrt{6}, 1)$.

(c) (10 points) Find a point on the surface $S$ so that the tangent plane to $S$ goes through the point $(3, 0, 0)$.

(d) (3 points) Describe the set of points on the surface $S$ so that the tangent plane to $S$ goes through the point $(3, 0, 0)$. 
5. (25 points) Find the equation of the tangent line to the curve of intersection of the surface

\[ z = 2x^3 + xy^2 + 5x^2 + y^2 \]

and the sphere

\[ x^2 + y^2 + z^2 = 2 \]

at the point (0,1,1).
6. (25 points) Find the maximal and minimal values of \( f(x, y, z) = x^2y^2 \) on the unit disk \( D = \{(x, y) \mid x^2 + y^2 \leq 1\} \). Why do these extrema exist?
7. (25 Points) Assume a particle moves along the curve given by the path 

\[ \vec{c}(t) = (\cos t, \sin t, \sin(\frac{t}{2})) \]

(a) Find the velocity and speed of the particle at time \( t = \frac{\pi}{2} \).

(b) Find the equation of the tangent line to the curve at time \( t = \frac{\pi}{2} \).

(c) Find the acceleration of the particle at time \( t = \frac{\pi}{2} \).

(d) Is the particle speeding up or slowing down at time \( t = \frac{\pi}{2} \)? Justify your answer.

(e) Give an integral for the arc length of the curve \( \vec{c}(t) \) between \( t = 0 \) and \( t = \frac{\pi}{2} \). You do NOT need to evaluate this integral.
8. (25 points) Let \( \vec{F}(x, y, z) = (x^2, x^2y, z + zx) \) be a vector field in \( \mathbb{R}^3 \).

Recall that the operator \( \nabla \) is given by the vector \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \)

(a) Show that \( \vec{c}(t) = \left( \frac{1}{1-t}, 0, \frac{e^t}{1-t} \right) \) is a flow line for \( \vec{F} \) where \( t \neq 1 \).

(b) Find the divergence of \( \vec{F} \), \( \nabla \cdot \vec{F} \).

(c) Find the curl of \( \vec{F} \), \( \nabla \times \vec{F} \).